Math 115, Second Midterm Questions.

Q1. Given \( f(x) = \frac{x^2 + 2x - 4}{x + 5} \).

Find the Taylor’s expansion of \( f(x) \) at point \( x_0 = -1 \). Write the interval of convergence in each case.

Q2. Evaluate a) \( \int_0^2 (|2x| + |x - 1| + \sqrt{4x - x^2}) \, dx \)  

b) \( \int_{-2}^2 (\ln \pi + x) \sqrt{4 - x^2} \, dx \).

(Explain your steps)

Q3. Let \( ABC \) be a triangle right-angled at \( C \) and having area \( 24 \text{cm}^2 \).

Find the maximum area of a rectangle inscribed in the triangle if one side of the rectangle lies along the hypotenuse, \( AB \).

Q4. Evaluate

a) \( \lim_{x \to 0} (1 + 2x + x^2)^\frac{1}{x} \),  
b) \( \lim_{x \to \infty} x(e^{\frac{1}{x}} - 1)^\frac{1}{x} \),  
c) \( \frac{d}{dx}(\sin x)^{\sqrt{x}} \)

Q5. Sketch the graph of the function \( f(x) = \frac{x^2 - 4}{x^2 - 1} \). Describe your answer.

Q6. Using definition of hyperbolic functions, prove that \( \sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}) \).