Math 115-01 Homework 2.

Q1. a) Does Intermediate Value Theorem apply to the function 
\[ f(x) = \frac{x^2 - 3}{x^2 + x - 2} \] on the interval \([0, 2]\)?

b) Show that the equation \(x^5 + 2x - 2 = 0\) has exactly one root on the interval 
\([0, 1]\).

Q2. Consider the curve \(x^\frac{2}{3} + y^\frac{2}{3} = 5\).

a) Verify that the point \(P_0 = (8, 1)\) lies on the curve.

b) Find the tangent line to the curve at point \(P_0\).

Q3. Find \(f'(-1)\), if it exists, by using the definition of derivative, where 
\[ f(x) = |x + 1| + 2 \]

Q4. Let \(f(x) = x^3 - 9x^2 + 24x + 30\). Find 

a) Local extrema 

b) The intervals of increase and decrease.

Q5. Let \(f(x) = x^3 - 6x^2 + 9x - 1\) be given on \([0, 5]\). Find, 

a) Absolute extrema on \([0, 5]\) 

b) The intervals of increase, decrease.

Q6. If \(\tan(xy^3) + \sqrt{y} = 2\). Find \(\frac{dy}{dx}\) at \((\frac{\pi}{4}, 1)\).

Q7. If \(g(x) = xf(x^2)\) and \(f(4) = 2, f'(4) = 1, f''(4) = -1\), then find \(g'(2)\).

Q8. Find the tangent line at \((-1, 3)\) to the curve \(y^4 - 2x^2y^3 - 27 = 0\).

Q9. Evaluate \(\lim_{x \to \infty} (x\sqrt{x^2 + 1} - x^2)\).

Q10. Evaluate a) \(\lim_{x \to 0} \frac{\sin(2x) - 2x}{x^3}\) 

b) \(\lim_{x \to \infty} (1 - 5x^2)\sin(\frac{1}{x^2})\).

Q11. Suppose that \(f'(4) = 3, g'(4) = 7, g(4) = 4\) and \(g(x) \neq 4\) for \(x \neq 4\). Find, 

a) \(\lim_{x \to 4} \frac{f(x) - f(4)}{\sqrt{x-2}}\) 

b) \(\lim_{x \to 4} \frac{f(x) - f(4)}{g(x) - g(2)}\).
Q12. Let

\[ f(x) = \begin{cases} 
  x + 2x^2, & x \neq 0, \\
  0, & x = 0.
\end{cases} \]

a) Is \( f \) continuous at \( x = 1 \)?

b) Is \( f \) differentiable at \( x = 1 \)?

Q13. Using the Mean Value Theorem, show that

a) \( \sin x < x \) for all \( x > 0 \),

b) \( \sqrt{1 + x} < 1 + \frac{x}{2} \) for all \( -1 \leq x < 0 \).

Q14. Find the coordinates of points on the curve \( y = \frac{x+1}{x+2} \) where the tangent line is parallel to the line \( y = 4x \).