

Table of z-Transform Pairs

$x[n] = \mathcal{Z}^{-1}\{X(z)\} = \frac{1}{2\pi j} \oint X(z)z^{n-1}dz$		\longleftrightarrow	$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$	ROC
transform	$x[n]$	\longleftrightarrow	$X(z)$	R_x
time reversal	$x[-n]$	\longleftrightarrow	$X(\frac{1}{z})$	$\frac{1}{R_x}$
complex conjugation	$x^*[n]$	\longleftrightarrow	$X^*(z^*)$	R_x
reversed conjugation	$x^*[-n]$	\longleftrightarrow	$X^*(\frac{1}{z^*})$	$\frac{1}{R_x}$
real part	$\Re\{x[n]\}$	\longleftrightarrow	$\frac{1}{2}[X(z) + X^*(z^*)]$	R_x
imaginary part	$\Im\{x[n]\}$	\longleftrightarrow	$\frac{1}{2j}[X(z) - X^*(z^*)]$	R_x
time shifting	$x[n - n_0]$	\longleftrightarrow	$z^{-n_0}X(z)$	R_x
scaling in \mathcal{Z}	$a^n x[n]$	\longleftrightarrow	$X(\frac{z}{a})$	$ a R_x$
downsampling by N	$x[Nn], N \in \mathbb{N}_0$	\longleftrightarrow	$\frac{1}{N} \sum_{k=0}^{N-1} X(W_N^k z^{\frac{1}{N}})$ $W_N = e^{-j\frac{2\pi}{N}}$	R_x
linearity	$ax_1[n] + bx_2[n]$	\longleftrightarrow	$aX_1(z) + bX_2(z)$	$R_x \cap R_y$
time multiplication	$x_1[n]x_2[n]$	\longleftrightarrow	$\frac{1}{2\pi j} \oint X_1(u)X_2(\frac{z}{u})u^{-1}du$	$R_x \cap R_y$
frequency convolution	$x_1[n] * x_2[n]$	\longleftrightarrow	$X_1(z)X_2(z)$	$R_x \cap R_y$
delta function	$\delta[n]$	\longleftrightarrow	1	$\forall z$
shifted delta function	$\delta[n - n_0]$	\longleftrightarrow	z^{-n_0}	$\forall z$
step	$u[n]$	\longleftrightarrow	$\frac{z}{z-1}$	$ z > 1$
	$-u[-n - 1]$	\longleftrightarrow	$\frac{z}{z-1}$	$ z < 1$
ramp	$nu[n]$	\longleftrightarrow	$\frac{z}{(z-1)^2}$	$ z > 1$
	$n^2u[n]$	\longleftrightarrow	$\frac{z(z+1)}{(z-1)^3}$	$ z > 1$
	$-n^2u[-n - 1]$	\longleftrightarrow	$\frac{z(z+1)}{(z-1)^3}$	$ z < 1$
	$n^3u[n]$	\longleftrightarrow	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$ z > 1$
	$-n^3u[-n - 1]$	\longleftrightarrow	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$ z < 1$
	$(-1)^n$	\longleftrightarrow	$\frac{z}{z+1}$	$ z < 1$
exponential	$a^n u[n]$	\longleftrightarrow	$\frac{z}{z-a}$	$ z > a $
	$-a^n u[-n - 1]$	\longleftrightarrow	$\frac{z}{z-a}$	$ z < a $
	$a^{n-1} u[n - 1]$	\longleftrightarrow	$\frac{1}{z-a}$	$ z > a $
	$na^n u[n]$	\longleftrightarrow	$\frac{az}{(z-a)^2}$	$ z > a $
	$n^2 a^n u[n]$	\longleftrightarrow	$\frac{az(z+a)}{(z-a)^3}$	$ z > a $
	$e^{-an} u[n]$	\longleftrightarrow	$\frac{z}{z-e^{-a}}$	$ z > e^{-a} $
exp. interval	$\begin{cases} a^n & n = 0, \dots, N - 1 \\ 0 & \text{otherwise} \end{cases}$	\longleftrightarrow	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$ z > 0$
sine	$\sin(\omega_0 n) u[n]$	\longleftrightarrow	$\frac{z \sin(\omega_0)}{z^2 - 2 \cos(\omega_0)z + 1}$	$ z > 1$
cosine	$\cos(\omega_0 n) u[n]$	\longleftrightarrow	$\frac{z(z - \cos(\omega_0))}{z^2 - 2 \cos(\omega_0)z + 1}$	$ z > 1$
	$a^n \sin(\omega_0 n) u[n]$	\longleftrightarrow	$\frac{za \sin(\omega_0)}{z^2 - 2a \cos(\omega_0)z + a^2}$	$ z > a$
	$a^n \cos(\omega_0 n) u[n]$	\longleftrightarrow	$\frac{z(z - a \cos(\omega_0))}{z^2 - 2a \cos(\omega_0)z + a^2}$	$ z > a$
differentiation in \mathcal{Z}	$nx[n]$	\longleftrightarrow	$-z \frac{dX(z)}{dz}$	R_x
integration in \mathcal{Z}	$\frac{x[n]}{n}$	\longleftrightarrow	$-\int_0^z \frac{X(z)}{z} dz$	R_x
	$\frac{\prod_{i=1}^m (n-i+1)}{a^m m!} a^m u[n]$	\longleftrightarrow	$\frac{z}{(z-a)^{m+1}}$	

Note:

$$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

Table of Laplace Transform Pairs

$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \lim_{T \rightarrow \infty} \int_{c-jT}^{c+jT} F(s)e^{st} ds$		$\xleftrightarrow{\mathcal{L}}$	$F(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{+\infty} f(t)e^{-st} dt$	
transform	$f(t)$	$\xleftrightarrow{\mathcal{L}}$	$F(s)$	
complex conjugation	$f^*(t)$	$\xleftrightarrow{\mathcal{L}}$	$F^*(s^*)$	
time shifting	$f(t-a) \quad t \geq a > 0$	$\xleftrightarrow{\mathcal{L}}$	$a^{-as} F(s)$	
	$e^{-at} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$F(s+a)$	frequency shifting
time scaling	$f(at)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{ a } F\left(\frac{s}{a}\right)$	
linearity	$af_1(t) + bf_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$aF_1(s) + bF_2(s)$	
time multiplication	$f_1(t)f_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$F_1(s) * F_2(s)$	frequency convolution
time convolution	$f_1(t) * f_2(t)$	$\xleftrightarrow{\mathcal{L}}$	$F_1(s)F_2(s)$	frequency product
delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1	
shifted delta function	$\delta(t-a)$	$\xleftrightarrow{\mathcal{L}}$	e^{-as}	exponential decay
unit step	$u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$	
ramp	$tu(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$	
parabola	$t^2 u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2}{s^3}$	
n -th power	t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s^{n+1}}$	
exponential decay	e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$	
two-sided exponential decay	$e^{-a t }$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2a}{a^2 - s^2}$	
	te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$	
	$(1-at)e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{(s+a)^2}$	
exponential approach	$1 - e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$	
sine	$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2 + \omega^2}$	
cosine	$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2 + \omega^2}$	
hyperbolic sine	$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2 - \omega^2}$	
hyperbolic cosine	$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2 - \omega^2}$	
exponentially decaying sine	$e^{-at} \sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2 + \omega^2}$	
exponentially decaying cosine	$e^{-at} \cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2 + \omega^2}$	
frequency differentiation	$tf(t)$	$\xleftrightarrow{\mathcal{L}}$	$-F'(s)$	
frequency n -th differentiation	$t^n f(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^n F^{(n)}(s)$	
time differentiation	$f'(t) = \frac{d}{dt} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s) - f(0)$	
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2 F(s) - sf(0) - f'(0)$	
time n -th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$	
time integration	$\int_0^t f(\tau) d\tau = (u * f)(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s} F(s)$	
frequency integration	$\frac{1}{t} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u) du$	
time inverse	$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s) - f^{-1}}{s}$	
time differentiation	$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \dots + \frac{f^{-n}(0)}{s}$	